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VASAVI COLLEGE OF ENGINEERING *(Autonomous)*, HYDERABAD

B.E. (CBCS) VI-Semester Advanced Supplementary Examinations, July-2019

Probability and Statistics for Engineers

(Open Elective-VII)

Time: 3 hours Max. Marks: 70

Note: Answer ALL questions in Part-A and any FIVE from Part-B

Part-A (10 × 2 = 20 Marks)

1. State the properties of the cumulative distribution function of a two-dimensional RV (X,Y)
2. If $f(x,y) = k(1-x-y), 0 < x,y < \frac{1}{2}$, is a joint density function, find k
3. If X and Y are independent random variables with density functions $f_x(x) = \frac{8}{x^3}, x > 2$ and $f_y(y) = 2y, 0 < y < 1$ respectively and Z=XY, find E(Z)
4. Define (i) Joint Characteristic function (ii) Joint Moment generating function of two-dimensional RV
5. When we sample from an infinite population, what happens to the standard error if the mean of the sample size is (a) increased from 50 to 200 (b) decreased from 225 to 25
6. $x_1, x_2, \ldots\ldots x_n$ is a random sample from a normal population N(μ,1). Show that $t = \frac{1}{n}\sum_{i=1}^{n} x_i^2$ is an unbiased estimator of $1 + \mu^2$
7. Derive the relation between residual sum of squares and least square estimators.
8. The summary statistics with x=width and y=height are given below

   $n = 50, \bar{x} = 88.34, \bar{y} = 305.58, S_{xx} = 7239.22, S_{xy} = 17840.1, S_{yy} = 66975.2$ find the least square line for predicting height from width
9. Show that if X and Y are independent RVs, then E(Y/X)=E(Y) and E(X/Y)=E(X)
10. What is the value of the finite population correction factor in the formula for $\sigma_{\bar{X}}^2$ when (i) n=5 and N=250 (ii) n=100 and N=5000

Part-B (5 × 10 = 50 Marks)

(All sub-questions carry equal marks)

11. a) A two-dimensional RV (X,Y) have a bivariate distribution given by $P(X = x, Y = y) = \frac{1}{27}(2x + y)$ where x and y can assume only the integer values 0,1 and 2. Find the marginal distributions of X and Y and the conditional distribution of Y for X=x

    b) Joint distribution of X and Y is given by: $f(x,y) = 4xye^{-(x^2+y^2)}; x \geq 0, y \geq 0$. Test whether X and Y are independent. For the above joint distribution, find the conditional density of X given Y=y

12. a) Let X and Y be two random variables each taking three values -1, 0, and 1, and having the joint probability distribution

Y \ X	-1	0	1	Total
-1	0	.1	.1	.2
0	.2	.2	.2	.6
1	0	.1	.1	.2
Total	.2	.4	.4	1.0

(i) Show that X and Y have different expectations
(ii) Prove that X and Y are uncorrelated
(iii) Find Var (X) and Var (Y)
(iv) Given that Y=0, what is the conditional probability distribution of X
(v) Find Var (Y/ X= -1)

Contd…2

b) If (X, Y) is uniformly distributed over the semicircle bounded by $y = \sqrt{1 - x^2}$ and y=0, find E(X and E(Y/X). Also verify the E{E(X/Y)}=E(X) and E{E(Y/X)}=E(Y)

13. a) A random sample $(X_1, X_2, X_3, X_4, X_5)$ of size 5 is drawn from a normal Population with unknown mean μ. Consider the following estimators to estimate μ:

$$(i)\ t_1 = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5}\ (ii)\ t_2 = \frac{X_1 + X_2}{2} + X_3\ (iii)\ t_1 = \frac{2X_1 + X_2 + \lambda X_3}{3}$$

where λ is such that t_3 is an unbiased estimator of μ.

Find λ. Are t_1 and t_2 unbiased? State giving reasons, the estimator which is best among t_1, t_2 and t_3.

b) One process of making green gasoline takes biomass in the form of sucrose and converts it into gasoline using catalytic reactions. At one step in a pilot process, the output includes carbon chains of length 3. Fifteen runs with same catalyst produced the yields (gal)

5.57 5.76 4.18 4.64 7.02 6.62 6.33 7.24 5.57 7.89 4.67 7.24 6.43 5.59 5.39

Treating the yields as a random sample from a normal population

(a) Obtain the maximum likelihood estimates of the mean yield and the variance
(b) Obtain the maximum likelihood estimate of the coefficient of variation

14. a) The following data pertain to the growth of a colony of bacteria in a culture medium:

Days since inoculation x	3	6	9	12	15	18
Count y	115000	147000	239000	356000	579000	864000

Fit an exponential curve to the given data. Obtain the bacteria count at the end of 20 days from the curve.

b) Twelve specimens of cold-reduced sheet steel, having different copper contents and annealing temperatures, are measured for hardness with the following results

Hardness (Rockwell 30-T)	Copper content (%)	Annealing Temperature (degrees F)
78.9	0.02	1000
65.1	0.02	1100
55.2	0.02	1200
56.4	0.02	1300
80.9	0.10	1000
69.7	0.10	1100
57.4	0.10	1200
55.4	0.10	1300
85.3	0.18	1000
71.8	0.18	1100
60.7	0.18	1200
58.9	0.18	1300

Fit an equation of the form$y = b_0 + b_1x_1 + b_2x_2$, where x_1 represents the copper content and x_2 represents the annealing temperature and y represents the hardness.

15. a) Trains X and Y arrive at a station at random between 8 A.M> and 8.20 A.M. Train A stops for 4 min. and train B stops for 5 min. assuming that the trains arrive independently of each other, find the probability that (i) X will arrive before Y, (ii) the trains will meet and (iii) X arrived before Y, assuming that they met.

Contd…3

) If the joint density function of (X,Y) is given by $f(x, y) = 2 - x - y, 0 \le x, y \le 1$ find E(X), E(Y), Var(X), Var(Y) and ρ_{xy}

. a) The sample values of population with p.d.f. $f(x) = (1 + \theta)x^{\theta}, 0 < x < 1, \theta > 0$ are given: 0.46, 0.38, .061, 0.82, 0.59, 0.53, 0.72, 0.44, 0.59 and 0.60. Find the estimate of θ by (i) method of moments (ii) maximum likelihood estimation

b) The following are measurements of the air velocity and evaporation coefficient of burning fuel droplets in an impulse engine

Air velocity (cm/s) x	20	60	100	140	180	220	260	300	340	380
Evaporation coefficient (mm^2/s)	0.18	0.37	0.35	0.78	0.56	0.75	1.18	1.36	1.17	1.65

Fit a straight line to these data by the method of least squares and use it to estimate the evaporation coefficient of a droplet when the air velocity is 190 cm/s

17. Answer any *two* of the following:

a) Let $f(x, y) = 8xy, 0 < x < y < 1; f(x, y) = 0$ elsewhere. Find (a) E(Y/X=x) (b)E(X/Y=y), (c) Var(Y/X=x)

b) The following are data on the drying time of a certain varnish and the amount of an additive that is intended to reduce the drying time:

Amount of varnish additive (grams) x	0	1	2	3	4	5	6	7	8
Drying time (hours) y	12	10.5	10	8	7	8	7.5	8.5	9

Fit a second degree polynomial by the method of least squares. Use the result to predict the drying time of the varnish when 6.5 grams of additive is being used.

(from remaining)

c) Given $f_{xy}(x, y) = cx(x - y), 0 < x < 2, -x < y < x$, and 0 elsewhere (a) evaluate c (b) find $f_x(x)$ (c) $f_{y/x}(y/x)$ and (d) $f_y(y)$

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